PHYS393: Statistical and Low Temperature Physics

Tutorial 1

1. A model thermodynamic assembly has single-particle states with energies 0, ϵ, 2ϵ, 3ϵ, 4ϵ, 5ϵ, 6ϵ... The assembly has four distinguishable (localised) particles and a total energy of 6ϵ.

   (i) Identify the nine possible distributions.

   (ii) Evaluate the total number of accessible microstates, Ω.

   (iii) Evaluate the mean populations of each of the energy states.

2. An unbiased coin is tossed N times. The total number of different sequences of heads and tails is Ω. The number of different sequences that contain given numbers of H heads and T tails is $t(H, T)$.

   (i) Write an expression for Ω.

   (ii) Write an expression for $t(N/2, N/2)$.

   (iii) Show that if N is large enough for Stirling's approximation to be valid, then $t(N/2, N/2) = \Omega$. Comment on the significance of this result.

   Hint: Evaluate $\ln t(N/2, N/2)$, and compare with $\ln \Omega$.

3. The energy levels of a localised particle are 0, ϵ and 2ϵ. The middle level is doubly degenerate (two quantum states have the same energy) and the other levels are singly degenerate.

   (i) Write down and simplify the partition function Z.

   (ii) By summing over energy states, find an expression for the total energy U for a collection of N particles, as a function of temperature T. Show that the result you obtain in this way is the same as that obtained from:

   $$U = N\frac{d \ln Z}{d\beta},$$

   where k is Boltzmann's constant, and $\beta = -1/kT$. Find the limiting energy as $T \to 0$, and as $T \to \infty$; explain these limits in physical terms.

   (iii) Derive an expression for the heat capacity at constant volume, C_V.

   (iv) Starting from the Helmholtz free energy (expressed in terms of the partition function), find an expression for the entropy, S, of the system as a function of temperature. Find limiting expressions for S when $T \to \infty$ and when $T \to 0$. Sketch a plot showing the variation of entropy with temperature.